

THE CALGARY MATHEMATICAL ASSOCIATION
 39th JUNIOR HIGH SCHOOL MATHEMATICS CONTEST
 APRIL 29, 2015

NAME: _____ GENDER: _____
PLEASE PRINT (First name Last name) (optional)

SCHOOL: _____ GRADE: _____
(9,8,7,...)

- You have 90 minutes for the examination. The test has two parts: PART A — short answer; and PART B — long answer. The exam has 9 pages including this one.
- Each correct answer to PART A will score 5 points. You must put the answer in the space provided. No part marks are given. PART A has a total possible score of 45 points.
- Each problem in PART B carries 9 points. You should show all your work. Some credit for each problem is based on the clarity and completeness of your answer. You should make it clear why the answer is correct. PART B has a total possible score of 54 points.
- You are permitted the use of rough paper. Geometry instruments are not necessary. References including mathematical tables and formula sheets are **not** permitted. Simple calculators without programming or graphic capabilities **are** allowed. Diagrams are not drawn to scale. They are intended as visual hints only.
- Hint: Read all the problems and select those you have the best chance to solve first. You may not have time to solve all the problems.

MARKERS' USE ONLY	
PART A _____ × 5	
B1	
B2	
B3	
B4	
B5	
B6	
TOTAL (max: 99)	

**BE SURE TO MARK YOUR NAME AND SCHOOL
 AT THE TOP OF THIS PAGE.**

THE EXAM HAS 9 PAGES INCLUDING THIS COVER PAGE.

**Please return the entire exam to your supervising teacher
 at the end of 90 minutes.**

PART A: SHORT ANSWER QUESTIONS (Place answers in the boxes provided)

A1 At a bus station, a bus leaves at 8:00 am and a new bus leaves every 7 minutes after that. At what time does the first bus after 9:00 am leave?

A1

A2 If we mix one litre of lemonade that contains 4% lemon with two litres of lemonade that contains 10% lemon, what is the percentage of lemon in the resulting three litre mixture?

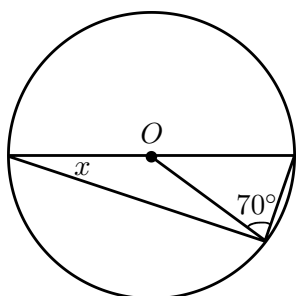
A2

A3 At the swimming pool last week, on each day there were ten fewer people than twice the number of people on the previous day. There were 130 people at the pool on Friday. How many people were at the pool on the previous Tuesday?

A3

A4 Given the circle below with centre O , find the angle x in degrees.

A4



A5 Firmamint boxes of chocolates contain 17 with hard centres and 5 with soft centres. Sweetart boxes contain 7 with hard centres and 11 with soft centres. If I buy one Firmamint box, how many Sweetart boxes must I buy so that the total number of hard centres is equal to the total number of soft centres?

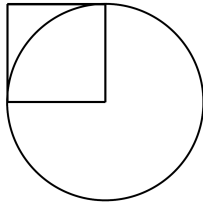
A5

A6 Sagal and Xi leave home at the same time to walk to the park which is 6 km away. Sagal walks at 1 km/hr for 2 km, then at 2 km/hr for 4 km. Xi walks at 1 km/hr for 4 km, then at 2 km/hr for 2 km. Sagal arrives at the park at noon. At what time does Xi arrive?

A6

A7 In the following figure the square has one corner in the centre of the circle and two sides are tangent to the circle. How many times larger is the area of the circle than the area of the square?

A7



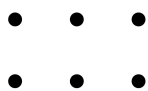
A8 Below, the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ are to be filled into the nine smaller squares so that every number is used exactly once. If the sum of each row and the sum of each column is at most 15, what must the value of x be?

A8

	x	
		7
8		

A9 How many triangles (with positive area) are there which have their three corners as points chosen from the 2×3 grid shown?

A9



B2 Archibald runs round a 300 metre circular race track at 7 km/hr, while Beauregard runs at 8 km/hr. Suppose they start at the same time at the same place, but run in opposite directions.

(a) How long in minutes will it be before they first meet?

(b) If they keep running, will they ever meet at the point where they started, and if so, after how many minutes?

B3 There are 2015 balls in 1000 boxes.

- (a) Each box contains 1, 2, or 3 balls.
- (b) The number of boxes containing exactly one ball is greater than 308.
- (c) The total number of balls in boxes containing more than one ball is greater than 1705.

How many boxes contain exactly 1, 2, and 3 balls, respectively?

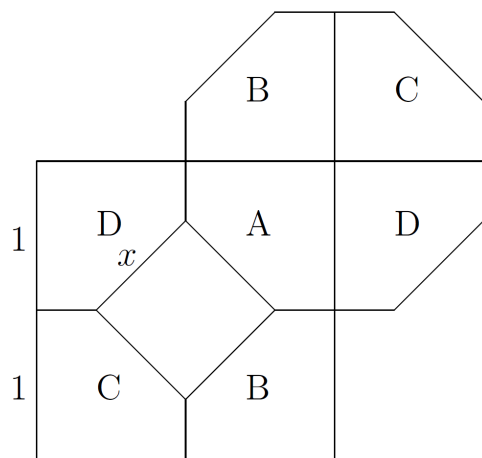
B4 A **preven number** is an integer that uses each digit in $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ at most once, both starts and ends with a single digit that is prime or even, and each pair of consecutive digits forms a two-digit number which is prime or even.

For example, 8347 is preven since its first digit is even, its last digit is prime, and any two consecutive digits (83, 34, 47) are either even or prime. On the other hand, 8743 is not preven since 87 is neither even nor prime. The number 8343 is also not preven since it has a repeated digit.

(a) Find a four-digit preven number larger than 8347. The larger your four-digit preven number is, the more marks you may earn.

(b) Find a preven number which is as large as possible. The larger your number is, the more marks you may earn.

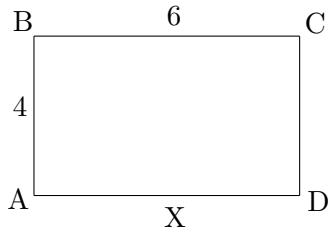
B5 A square with edge length 2 is cut into five pieces: a square of edge length x , and four congruent pieces, A, B, C, and D which are reassembled to form an octagon which is regular, that is, has all its eight edges equal in length.



(a) What is x ?

(b) Which piece has larger area: the square with edge length x or the piece labelled by A?

B6 Ellie is on her side of the tennis court (which is a 4 metres by 6 metres rectangle ABCD), practising serving from the midpoint X of the baseline AD. When there are three balls lying in her court she walks in straight lines to pick them up, from X to one ball, then to a second ball, then to the third ball and back to X. For example, if there were two balls at B and one at C, she could travel XBBCX for a total distance of $5+0+6+5=16$ metres, or she could go XBCBX for a distance of $5+6+6+5=22$ metres.



(a) Suppose the three balls are at points A, B and C. What is the shortest distance Ellie could walk to pick up the three balls? What is the longest distance Ellie could walk to pick up the three balls?

(b) Find places on the court for the three balls to be located so that the ratio

$$\frac{\text{longest distance Ellie could walk}}{\text{shortest distance Ellie could walk}}$$

is at least 1.5. The larger a ratio you find, the better your mark will be. (For extra credit, prove that your ratio is as large as possible.)